Multi-Modal Localization Algorithm for Catheter Interventions

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Abstract—Localization of steerable catheters in minimally invasive surgery is critical with respect to patient safety, surgeon manipulation, and procedural efficacy. While there are many potential benefits to patients including shorter recovery times, less tissue trauma, and lower infection rates than traditional surgeries, localization of surgical tools is still an area of much research. Current technology offers several sensory modalities. However, each system has drawbacks which do not provide a clear best practice. This research focuses on incorporating redundant commonplace surgical sensing technologies to reduce the likelihood of errors, failures, or inherent sensor characteristics causing harm to the patient and/or surgeon while providing accurate localization. Dual particle filters are implemented using both a fluoroscopic-like stereo imaging system and an electromagnetic pose sensor for measurement updates in a prototype catheter testbed. A previously developed catheter model is modified to increase accuracy in the particle filter outputs which are combined using a weighted average based on each filter’s particle statistics. Experimental results implementing the combined particle filter multi-modality algorithm in feedback control validates the algorithm’s ability to provide accurate localization in a surgical setting while overcoming sensor limitations and possible failure modes.

I. INTRODUCTION

The application of robotic technology in the medical field has grown tremendously since the advent of minimally invasive surgery (MIS) in the late 1980’s \cite{1}. These types of procedures promise many benefits to patients compared to traditional surgeries including lower infection risk, less tissue damage, and faster recovery times \cite{2}. These come at a cost however to the surgeon in that long thin application specific tools are required where the field of view is obstructed. Additional disadvantages include the loss of hand eye coordination, loss of direct force feedback from the tissue, and constrained motion of the instruments \cite{1}, \cite{2}. While minimally invasive robotic surgery offers much potential the application is inherently complex demanding more research and development \cite{3}.

A. MIS Direct Sensing Systems

Localization of surgical tools relative to the targeted anatomy \textit{in-vivo} is a critical aspect of any robotic MIS system. Due to the obstruction from direct view, this localization provides feedback for maneuvering toward the target without causing unnecessary damage to surrounding tissue. The quality and method of localization can influence the surgeon’s control decisions based on expertise level and/or control decisions within the robotic system’s automation process. Steerable catheters provide many benefits for minimally invasive procedures such as ablation of atrial fibrillation \cite{2}. With few moving parts these devices are easy to miniaturize with small diameters at low cost. This makes catheters ideal for procedures on organs which can be accessed through the vascular system.

Typical sensing modalities for catheter interventions include ultrasound, magnetic resonance imaging (MRI), electromagnetic pose sensors (EPS), or fluoroscopy (X-rays) \cite{4}. Ultrasound is inexpensive, nontoxic, and very safe to the patient. However, the image quality is subject to the operator’s skill and images can be highly distorted as a result of shadows, reflections, speckle, and reverberations. MRI provides high quality soft tissue resolution in a non-toxic environment. However, they have limited space, high magnetic fields which makes instrumentation difficult, and are very expensive compared to other modalities \cite{4}. Fluoroscopy is easy to use, versatile, and widely available. Soft tissue resolution is poor but catheters are easily distinguishable allowing for real-time tracking. The main disadvantage is the exposure of the patient and surgeon to radiation. EPS systems use a magnetic field transmitter outside the body and receiving sensors mounted internal to the catheter to calculate the catheter’s pose. These sensors are non-toxic, reasonably accurate, and relatively inexpensive. However, these systems are susceptible to drift and distortion in the presence of ferrous materials and other electromagnetic fields in a surgical setting.

B. Related Work

Many researchers across the medical and engineering fields have reported on the results of using different modalities for catheter localization, guidance, and control. Several investigations have focused on using imaging for localization and closed loop control from surgical settings with catheters and flexible needles to more general applications in continuum robotics \cite{5}–\cite{8}. With fluoroscopy becoming ubiquitous in contemporary surgery, much research has been devoted to mitigating the risk of radiation exposure to both surgeon and patient \cite{9}–\cite{13}.

An indirect method of estimating the resulting pose of a steerable catheter is using known inputs with a predictive model. Much research has been devoted to developing models that give accurate predictive results \cite{8}, \cite{13}–\cite{16}. However, due to the difficulty in modeling the effects of hysteresis in a flexible catheter, these models can still generate significant errors between predicted and actual poses.

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Recent research has been directed at the use of multiple sensors and state estimation algorithms. In [17], a kinematic catheter model was successfully used in a particle filter implementation for 3D pose estimation and tracking of the catheter tip relative to a heart phantom. The measurement update comes from an intracardiac echocardiography catheter (ICE) where the control input is provided by an EPS sensor. A sensor fusion algorithm was developed in [18] where an EPS is used in conjunction with an optical tracking system. Through the implementation of an unscented Kalman filter the system provides localization even in the event of an optical marker occlusion or magnetic field distortion. Additionally, in [19] a Kalman filter based sensor fusion algorithm was developed using a kinematic model and two EPS’s for reducing error in surgical needle deflection. And still in [20], position measurements of a continuum robot are fused with actuator inputs and a model prediction using an Extended Kalman Filter to estimate best estimate the pose and the applied force.

C. Contributions

This paper intends to provide the groundwork for developing real-time multi-modal catheter localization for closed-loop control in surgical interventions. The key contributions to the field include: (1) probabilistic integration of multiple types of redundant modalities, (2) modality fault and error mitigation, and (3) continued seamless operation during presence of error conditions. The focus is to use current sensing technologies, including an EPS system, a stereo camera imaging system in place of fluoroscopy, and a predictive catheter configuration model with measured inputs, in an intelligent way.

II. EXPERIMENTAL TESTBED

A testbed was constructed to investigate feedback control of continuum robotic manipulators or, more specifically, steerable catheters. The primary objective was to provide a system with a number of sensory sources representative of those currently available to clinicians. This allows the evaluation of localization and control strategies in the context of a realistic clinical environment. The testbed and associated equipment, shown in Figure 1, consists of a catheter manipulation system, measurement systems, and a data acquisition and management system.

A durable continuum manipulator prototype was constructed that provides similar actuation and response to commercial catheters while also being relatively easy and inexpensive to build. This manipulator, shown in Figure 2, is constructed from a flexible polymer with four, diametrically opposed, internal lumens. These supporting lumens provide passageways for the control wires, or tendons, made of monofilament wires attached to the distal cap. Through the actions of pulling on each of the control tendons the catheter can be manipulated and deformed to a desired tip pose.

The catheter is supported by a rigidly fixed proximal teflon sheave. The control tendons extend from the distal cap through the sheave to the Continuum Robotics Electromechanical System Testbed (CREST). This device provides tendon articulation (cable spool assembly) and insertion manipulation (screw drive) [21]. During typical procedures a clinician will sit at a workstation and input commands through various I/O channels in order to manipulate the catheter. The CREST takes these signals as input, converts them to motor rotation commands, and executes them in order to pull the control tendons and manipulate the catheter in the desired manner. Feedback motor control is implemented internally on each of the control wire and insertion axis motors. Current catheter procedures rely primarily on fluoroscopic imaging to provide feedback to the clinician. Due to the costs and radiation exposure of actual fluoroscopic systems, a stereo camera imaging system has been developed as a substitute. This system provides real-time images of the testbed catheter from two different viewing angles simultaneously. The stereo system is calibrated to the catheter fixture using standard techniques and the calibration grid shown. Standard image processing routines including threshold binarization and connected component analysis are used for segmentation, where the distal cap functions as a fiducial, to localize the catheter tip position in the workspace.
Electromagnetic localization systems provide the benefit of short range motion tracking without the radiation exposure of fluoroscopy. This testbed utilizes a trakSTAR system (Ascension Technologies, Burlington, VT) where the sensors instantly measure the transmitted field vectors and compute their real-time position and orientation relative to the transmitter. A sensor is placed through the center lumen of the prototype into the center of the distal cap and calibrated to the catheter.

III. CATHETER MODEL

A mechanics based model is used for catheter control and localization, mapping manipulated inputs to the output catheter endpoint position. The model, depicted in Figure 3, is based on one originally presented in [8] and later expanded in [22] and [14]. This model assumes constant curvature and no friction between the control tendons and supporting lumens. Modifications, however, have been made to accommodate the insertion axis degree of freedom and to allow additional degrees of freedom in the modeling parameters.

![Fig. 3. Catheter Model Configuration Parameters, including insertion curvature \( \kappa \), arc length \( s \), articulation angle \( \phi \), and orientation angle \( \beta \), and Control Tendon Inputs \( \Delta l_1 \) through \( \Delta l_4 \) and Insertion Axis Input \( \Delta l_5 \)](image)

The state vector \( \mathbf{x} \) represents the position coordinates of the catheter fiducial, corresponding to the endpoint fiducial frame \( \{f\} \), in the reference frame \( \{r\} \) as

\[
\mathbf{x} = \begin{pmatrix} X & Y & Z \end{pmatrix}^T
\]

where \( \{r\} \) is defined affixed to the end of the proximal sheave on the catheter fixture. The development of the model, estimating the state output based on the manipulated tendon inputs, is detailed as follows.

A. Motor Inputs to Configuration Space

The prototype catheter is manipulated through the control actions of the CREST drive motors. These manipulated inputs are represented as

\[
\mathbf{\theta} = \begin{pmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5 \end{pmatrix}^T
\]

where \( \theta_1 \) through \( \theta_4 \) are the angular articulation axes positions corresponding to the tendon control wires and \( \theta_5 \) is the angular position of the insertion axis lead screw. The tendon and insertion displacements depicted in Figure 3 are represented by the vector

\[
\mathbf{y} = \begin{pmatrix} \Delta l_1 & \Delta l_2 & \Delta l_3 & \Delta l_4 & \Delta l_5 \end{pmatrix}^T
\]

referred to as the catheter joint space. The relationship of motor inputs to joint space is given by

\[
\mathbf{y} = R_a \mathbf{\theta}
\]

where \( R_a \) is a diagonal gain matrix of articulation axes spool radii and the insertion axis lead screw pitch.

Assuming the catheter to be quasi-static, the authors in [8] derive moment balance, about the \( x \) and \( z \) axes, and force balance equations yielding

\[
\begin{pmatrix} K_{bx} & K_{bz} & K_a \end{pmatrix} \mathbf{q}_c = \begin{pmatrix} T_1d_1 - T_3d_3 \\ T_3d_4 - T_2d_2 \\ \sum_{i=1}^{4} T_i \end{pmatrix}
\]

where \( \mathbf{q}_c \) is the configuration state vector. This vector is defined as

\[
\mathbf{q}_c = \begin{pmatrix} \kappa_x & \kappa_z & \epsilon_a \end{pmatrix}^T
\]

where \( \kappa_x \) and \( \kappa_z \) are the signed neutral axis curvature components, about the \( x \) and \( z \) axes respectively, and \( \epsilon_a \) is the strain along the central axis. The parameters, \( d_1 \) through \( d_4 \), represent the offset perpendicular distance between the central axis and each tendon control wire. The tension induced moments are related to the neutral axis curvature components by the \( x \) and \( z \) axis bending stiffnesses, \( K_{bx} \) and \( K_{bz} \). The tendon tensions, \( T_1 \) through \( T_4 \), are related to the axial strain by the axial stiffness \( K_a \).

The resultant combined effect of the catheter’s axial compression, bending compression, and strain within each tendon are given by the following “conservation of strain” equations [8]

\[
\begin{pmatrix} \Delta l_1 \\ \Delta l_2 \\ \Delta l_3 \\ \Delta l_4 \end{pmatrix} = \begin{pmatrix} l_b \kappa_x d_1 + l_a \epsilon_a + \frac{1}{c_1} T_1 \\ -l_b \kappa_x d_2 + l_a \epsilon_a + \frac{1}{c_2} T_2 \\ -l_b \kappa_x d_3 + l_a \epsilon_a + \frac{1}{c_3} T_3 \\ l_b \kappa_x d_4 + l_a \epsilon_a + \frac{1}{c_4} T_4 \end{pmatrix}
\]

In these equations, \( l_b \) is the undeformed catheter length, \( l_a \) is the undeformed bending length, and coefficients \( c_1 \) through \( c_4 \) are defined as

\[
c = \frac{K_i}{l_i}
\]

where \( l_i \) is the tendon undeformed length and \( K_i \) is the tendon axial stiffness. If any of the tendons are in a slack condition then the corresponding coefficient is set to zero because slack tendons have no effect on the catheter configuration.

Assuming uniform strain along the entire length of the catheter and neglecting friction effects between the catheter and the sheave, a strain equation is derived for the catheter...
section contained within the proximal sheave and yields the following

$$l_b = l_a - \left( \frac{\Delta l_5}{1 - \epsilon_a} \right)$$  (9)

The unknown configuration space, $q_c$, can be found by first eliminating the tendon tensions in Equations 5 using Equations 7. Then, given a joint vector $y$ and assuming $\epsilon_a \ll 1$, an iterative solution is formulated by constructing a linear system of the form

$$M \dot{q}_c = b$$  (10)

where

$$M = \begin{bmatrix}
    l_b(c_1 d_1 - c_3 d_3) & l_b(c_4 d_4 - c_2 d_2) & \kappa_a + l_b \sum_{i=1} c_i \\
    \kappa_b + l_b(c_1 d_1 + c_3 d_3) & 0 & l_b(c_1 d_1 - c_3 d_3) \\
    0 & \kappa_b + l_b(c_2 d_2 + c_4 d_4) & l_b(c_4 d_4 - c_2 d_2)
\end{bmatrix}$$  (11)

and

$$b = \begin{pmatrix}
    \sum_{i=1} c_i \Delta l_i (c_1 d_1 d_1 - c_3 d_3 d_3) (c_4 d_4 d_4 - c_2 d_2 d_2)
\end{pmatrix}^T$$  (12)

This iterative approach recursively estimates $\dot{l}_b$ and $\dot{q}_c$ until a convergence criterion is met defined as the iteration when the difference between successive values of $\dot{\epsilon}_a$ fall below a tunable threshold value, $\delta$. For all experiments conducted, convergence occurred within 2 time steps. This algorithm, symbolized by $A(y)$, is summarized in Table I.

**Function $A(y)$**

- Set $\dot{\epsilon}_a = 0$ and calculate $\dot{l}_b$
- Construct $M$, $b$, and solve $\dot{q}_c = M^{-1} b$
- do
  - Set $\dot{\epsilon}_a = \dot{\epsilon}_a$ and calculate $\dot{l}_b$
  - Construct $M$ and solve $\dot{q}_c = M^{-1} b$
- while $|\dot{\epsilon}_a - \dot{\epsilon}_a| > \delta$
- if $\dot{\epsilon}_a <= 0$
  - Set $\dot{\epsilon}_a = 0$ and compute $\kappa_x$ and $\kappa_z$
- return $q_c = \dot{q}_c$

**TABLE I**

**JOINT TO CONFIGURATION FUNCTION, $A(y)$, ALGORITHM**

**B. Configuration to State Space**

In Figure 3, the catheter is shown in a general configuration described geometrically by the curvature $\kappa$, the arc length $s$ along the central axis, and the angle $\beta$ measured counter-clockwise from the $x$ axis to the projection of the catheter onto the $xz$ plane. These geometric parameters are defined in [23] as

$$\begin{pmatrix}
    s \\
    \kappa \\
    \beta
\end{pmatrix} = \begin{pmatrix}
    l_b(1 - \epsilon_a) \\
    \sqrt{\kappa_y^2 + \kappa_z^2} \\
    \arctan \frac{\kappa_z}{\kappa_y}
\end{pmatrix}$$  (13)

The coordinates of the catheter state vector are then derived in these geometric terms as

$$\begin{pmatrix}
    X \\
    Y \\
    Z
\end{pmatrix} = \begin{pmatrix}
    \frac{\cos \beta}{\kappa} (1 - \cos s \kappa) \\
    \frac{\sin \beta}{\kappa} (1 - \cos s \kappa)
\end{pmatrix}$$  (14)

Equations 13 and 14 are encapsulated in a configuration to joint space function symbolized by $H(q_c)$.

**C. Calibration**

Recorded images were used to measure catheter curvature and strain. Tendon displacements correlating to each image were determined from the CREST motor encoders. Combining the calibration data with Equations 5 and 7 and initial estimates of the parameters, a minimization routine converged to the values given in Table II.

**TABLE II**

**CALIBRATED CATHETER MODEL PARAMETERS**

<table>
<thead>
<tr>
<th>$l_a$</th>
<th>$K_a$</th>
<th>$K_{bz}$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>83.57 mm</td>
<td>761.25 N</td>
<td>1765.98 N-m$^2$</td>
<td>2.01 mm</td>
<td>2.01 mm</td>
<td>1.84 mm</td>
<td>2.26 mm</td>
<td>10.60 N/mm</td>
<td>15.44 N/mm</td>
<td>20.66 N/mm</td>
<td>12.83 N/mm</td>
</tr>
</tbody>
</table>

**IV. ENDPOINT CLOSED-LOOP CONTROL**

The control system provides a baseline trajectory for evaluating both measurement systems and the model furthering development of the multi-modal localization. Developing a robust control system which operates the catheter through trajectories similar to surgical procedures provides a framework for development of increasingly autonomous robotic surgery.

**A. Control Architecture**

The control system, depicted in Figure 4, utilizes a PI controller with State Command Feed-forward (CFF). The forward catheter model is implemented for state estimation based on the measured motor outputs. The inverse catheter model developed to allow the controller to operate in state space is as follows.
B. State to Configuration Space

Taking the catheter state \( x \) as input with the constant curvature assumption, the coordinates of point \( C \) in Figure 3 are derived geometrically as

\[
\begin{pmatrix}
X_C \\
Z_C
\end{pmatrix} = \begin{pmatrix}
\frac{X^2 + Y^2 + Z^2}{2(X + Y + Z)} \\
\frac{X^2 + Y^2 + Z^2}{2(X + Y + Z)}
\end{pmatrix} \begin{pmatrix}
X \\
Z
\end{pmatrix}
\] (15)

With these coordinate values known the articulation angle \( \phi \) is determined from

\[
\phi = \arctan\left(\frac{Y}{\sqrt{(X - X_C)^2 + (Z - Z_C)^2}}\right)
\] (16)

The geometric configuration parameters are then found as

\[
\begin{pmatrix}
s \\ \kappa \\ \beta
\end{pmatrix} = \begin{pmatrix}
\sqrt{\frac{X^2 + Y^2 + Z^2}{2(1 - \cos \phi)}} \\
\sqrt{\frac{2(1 - \cos \phi)}{X^2 + Y^2 + Z^2}} \\
\arctan\left(\frac{Z}{X}\right)
\end{pmatrix} \begin{pmatrix}
\phi \\
\phi
\end{pmatrix}
\] (17)

With 5 control inputs there is no unique solution for the configuration vector \( q_c \) without defining additional constraints. In [8], a minimum tension was maintained in the tendon control wires for any given configuration. However, because the tendon axial offsets vary (Table II), the constraint here is defined by the minimum induced torque \( \tau_{min} \) on the catheter with the minimum tension in the \( i \)-th tendon defined as

\[
T_i \geq \frac{1}{d_i} \tau_{min}
\] (18)

Setting \( \tau_{min} > 0 \) prevents tendon slack allowing for smooth transitions past the zero curvature point with no backlash.

Solving for the neutral axis curvature components from the last two rows of Equation 13 yields

\[
\begin{pmatrix}
\kappa_x \\
\kappa_z
\end{pmatrix} = -\kappa(1 - \epsilon_a) \begin{pmatrix}
\sin \beta \\
\cos \beta
\end{pmatrix}
\] (19)

Combining with Equation 5, enforcing minimum torque conditions, eliminating the tensions and neutral axis curvature components, and solving for the strain yields

\[
\epsilon_a = \frac{C\kappa + \tau_{min} \sum_{i=1}^{4} d_i^{-1}}{C\kappa + K_a}
\] (20)

where

\[
C = \frac{1}{d_m} K_{bz} |\sin \beta| + \frac{1}{d_n} K_{bz} |\cos \beta|
\] (21)

The indices \( m \) and \( n \) are used to identify the tendons on the concave or higher tension side of the central axis and are identified by the quadrant of \( \beta \). With \( \epsilon_a \) found, the curvature components \( \kappa_x \) and \( \kappa_z \) can be calculated from Equation 19, thereby, solving \( q_c \). This process is summarized by the function block \( H^{-1}(s) \) in Figure 4.

C. Configuration to State Space

From the minimum tension conditions enforced in the previous section in the development of Equation 20, the corresponding maximum tendon tension values are derived as

\[
\begin{pmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4
\end{pmatrix} = \begin{pmatrix}
\frac{1}{d_1} \tau_{min} + K_{bz} \kappa_x \\
\frac{1}{d_2} \tau_{min} - K_{bz} \kappa_x \\
\frac{1}{d_3} \tau_{min} - K_{bz} \kappa_z \\
\frac{1}{d_4} \tau_{min} + K_{bz} \kappa_z
\end{pmatrix}
\] (22)

where the maximum tensions occur in the tendons on the concave side of the catheter curvature dependent on \( \beta \).

The insertion axis displacement \( \Delta l_z \) is found by combining Equation 9 with the first row of Equation 13 yielding

\[
\Delta l_z = l_a(1 - \epsilon_a) - s
\] (23)

With the tendon tensions and \( q_c \) known, the joint space vector, \( y \) is found directly from Equations 7 and 23. This process is encapsulated by the function block \( A^{-1}(q_c) \) in Figure 4. The motor inputs are then found by inverting the gain matrix \( R_a \) from Equation 4 and solving for \( \theta \).
D. Experimental Results

Experimental results using EPS for feedback control are shown in Figure 5. The full 360° circular path is traversed in 120 seconds to help the controller reach steady state and maintain stability. The commanded trajectory, the measured states, and the model predicted state based on the measured CREST motor positions are shown in Figure 5(a). The 3D distance tracking error for the EPS is shown in Figure 5(b) together with the relative distance errors of the imaging system and model each with respect to the EPS output.

The efficacy of the control system is sufficient for evaluating both measurement systems and the model. The model is not as accurate as either measurement system over most of the trajectory. The relative model error, however, does approach and even fall below the relative error of the imaging system at certain locations. The relative model error peaks as \( \beta \) crosses the \( x \) and \( z \) axes when curvature control is passed between diametrically opposed control wires. This behavior indicates unmodeled, non-uniform friction effects between the control wires and the catheter lumens [8].

V. MULTI-MODAL LOCALIZATION

The localization algorithm implements a pair of particle filters, integrating each measurement system output with the model. The filter state outputs are merged using particle set statistics to estimate the state of the catheter.

A. Particle Filter

The particle filter implementation comes directly from [24] where each particle is a representation of an instance of the catheter state at a given time. The set of \( M \) particles, \( X_k \), describing possible states of the catheter at a time step \( k \) is represented as

\[
X_k = \{ (x_k)_1, (x_k)_2, \ldots, (x_k)_M \}
\]  

(24)

Particles are initially generated by sampling values from a zero mean Gaussian distribution and adding this “noise” to the catheter home position. A variance of 2 mm is based on data collected during the calibration process.

Particles evolve through the forward catheter model at each time step using the measured motor inputs as the control. Gaussian noise is added into the evolved particle where the variance is proportional to the motor input displacements relative to each particle’s previous state. Importance weights are calculated by inputting the Euclidean distance, between each evolved particle and the measured state, into a normal distribution function where the variance is sensor dependent.

The set of weights is given by

\[
W_k = \{ (w_k)_1, (w_k)_2, \ldots, (w_k)_M \}
\]  

(25)

and the variance for both the imaging system and EPS was set to 1 mm based on sensor calibration data. The algorithm draws with replacement \( M \) evolved particles from \( X_k \) where the probability of being drawn is based on the importance weight. This process tends to draw particles with higher likelihood or importance with respect to the measured state. In this manner, each particle filter simultaneously evolves and updates many instances of the state maintaining the particles with the highest probability of representing the “true” state at each time step.

B. State Estimation

The first filter (PF1) uses the EPS system for the measurement update while the second (PF2) uses the imaging system. The outputs of the particle filters are combined at each time step to produce the overall state estimate \( x_{SE} \) of the catheter endpoint by the following equation

\[
x_{SE} = \frac{\overline{w}_{PF1} x_{PF1} + \overline{w}_{PF2} x_{PF2}}{\overline{w}_{PF1} + \overline{w}_{PF2}}
\]  

(26)

The terms \( \overline{w}_{PF1} \) and \( \overline{w}_{PF2} \) represent the computed average of each filter’s particle set at each time step after resampling. Correspondingly, the terms \( \overline{w}_{PF1} \) and \( \overline{w}_{PF2} \) represent the average of each filter’s importance weight set after resampling. In this way, the confidence of each particle set is incorporated to produce a weighted average estimated state \( x_{SE} \).

C. Experimental Results

Data collected from the experiment depicted in Figure 5 was re-processed implementing the particle filter and state estimation algorithms. Each particle filter was initialized with 200 particles generated in the vicinity of the catheter’s home position. Two trials were run where Trial 1 processed the data without modification. Trial 2 modified the data to simulate sensor error and failure conditions. These conditions are (1) Image Outlier, (2) Imaging Off, and (3) EPS Drift. Condition (1) indicates something in the image, such as a reflection, briefly caused an erroneous measured state calculation. Condition (2) indicates that either or both fluoroscopes were shut off for a time. Condition (3) indicates the EPS measured state drifted as would happen in the presence of a ferrous material. An averaging filter was implemented on \( x_{SE} \) to reduce noise.

Shown in Figure 6(a) and (b) are the 3D particle filter and state estimation trajectories. In Figure 6(c) and (d) are the distance errors relative to the EPS measured state. Finally, shown in Figure 6(e) and (f) are the weight ratios used in the calculation of \( x_{SE} \). The state estimate transition from one particle filter to the other depends entirely on the importance weight average of each filter. The higher the weight average, the higher the probability that the measurement system agrees with the particles predicted by the model. Because the model output varies significantly over the trajectory, as seen by the error in Figure 5(b), and the measurement systems themselves have offsets and inconsistencies over the entire trajectory the switching in Figure 6(e) is expected. These plots validate the state estimation algorithm in completely rejecting the sensor failure conditions.

Finally, an experiment was performed to validate the efficacy of the state estimation algorithm for feedback control (Figure 4). The corresponding 3D particle filter and state estimator outputs, the relative distance errors with respect to the EPS measurement for correlation, and the weight ratios defining the calculation of \( x_{SE} \) are shown in Figure 7. The
results are similar to Figure 6 where the state estimation algorithm transitions from one particle filter output to the other over the course of the trajectory as expected.

VI. CONCLUSIONS AND FUTURE WORK

The main focus of the work presented is the development and evaluation of a multi-modal localization algorithm for surgical catheter interventions. A catheter model from prior literature was modified to predict the prototype’s output state to within a suitable range of the measurement systems to enable proof of concept experiments for the localization algorithm. Additionally, this model was implemented in a feedback control system to provide a means of evaluating the model, measurement systems, particle filters, and combined state estimation.

This state estimation comes from the weighted average of particle filter importance weights. Experimental results validated the dual particle filter implementations using the EPS and imaging system for measurement updates. The state estimation algorithm was shown to reject sensor errors and failures through simulated results using testbed data. Additionally, the algorithm was validated in providing seamless state feedback for real-time testbed catheter control.

Ongoing research will attempt to characterize the noise parameters over the catheter operating range. Further development will allow the particle filter noise parameters to
adjust to the current configuration of the model. Finally, evaluation of sensor error conditions using the state estimator for feedback control will be explored including development of a process to reduce radiation exposure.

![Control System Experimental Results with Commanded Fiducial Trajectory of 65mm Radius at 50mm Insertion Depth using the Estimated State for Feedback Control](image)

Fig. 7. Control System Experimental Results with Commanded Fiducial Trajectory of 65mm Radius at 50mm Insertion Depth using the Estimated State for Feedback Control

REFERENCES


