A HYBRID CONTINUUM-RIGID MANIPULATION APPROACH FOR ROBOTIC MINIMALLY-INVASIVE FLEXIBLE CATHETER BASED PROCEDURES

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ABSTRACT

In recent years, minimally-invasive surgical systems based on flexible robotic manipulators have met with success. One major advantage of the flexible manipulator approach is its superior safety characteristics as compared to rigid manipulators. However, their soft compliant structure, in combination with internal friction, results in poor position and force regulation which have limited their use to simpler surgical procedures. In this paper, we discuss a new approach to continuum robotic manipulation, interleaved continuum-rigid manipulation, which combines flexible, actively actuated continuum segments with small rigid-link actuators. The small rigid-link joints are interleaved between successive continuum segments and provide a redundant motion and error correction capability. We describe the overall approach including kinematic, design, and control considerations and investigate its performance using a one degree-of-freedom testbed and two degree-of-freedom planar simulation.

1. INTRODUCTION

While researchers have developed a variety of minimally-invasive surgical (MIS) robotic systems, the majority of MIS manipulation systems can be classified as either rigid-link manipulators, such as the Intuitive Surgical Da Vinci system [1], or flexible continuum manipulators, such as the Hansen Medical Artisan catheter system [2] or the Stereotaxis Niobe system [3]. One major advantage of the flexible manipulator approach is its superior safety characteristics as compared to rigid manipulators. A compliant structure, in combination with soft atraumatic construction, makes these manipulators much less likely to cause damage when they come in contact with tissue. For these reasons, flexible manipulators, including catheters, have become the dominant interventional tool in applications where safety is of particular concern, such as in intracardiac interventional procedures. While MIS systems based on flexible robotic manipulators have met with success, the very features which enable their superior safety characteristics have hindered their use in high performance manipulation tasks. Their soft compliant structure, in combination with the internal friction inherent to their design, results in poor position [4, 5] and force regulation, limiting their use to simpler surgical procedures.

While improvements in performance and dexterity would have benefits for a variety of flexible manipulator-based procedures, perhaps the most compelling category entails minimally-invasive cardiac interventional procedures. These procedures often take place in the chambers of a beating heart, requiring free space end-effector positioning and navigation. In many applications, the positioning accuracy and dexterity required exceeds that of currently available devices.

One such prototypical application is cardiac tissue ablation for the treatment of atrial fibrillation (AF). In the case of AF treatment, the ablation tip, located on the end of a flexible manually or robotically controlled catheter [2, 3, 6, 7], must be maneuvered around the pulmonary vein ostia while the tissue is
ablated, thereby achieving a conduction block of the aberrant electrical pathways. To achieve full isolation, the ablation lesion path must be both transmural and contiguous. To achieve this, the ablation tip must be positioned with sufficient contact force to achieve good energy transfer to the tissue while minimizing overall force application so as to prevent serious complications including cardiac tapenade and esophageal fistula, both of which are commonly fatal. Typical contact forces for this procedure range from 0.05 to 2.0 Newton [8]. In addition, achieving sufficient position control of the ablation tip such that a contiguous lesion can be formed has proven to be challenging, particularly for difficult to reach anatomical features such as the left superior and inferior pulmonary veins. While not well understood, positioning requirements on lesion placement accuracy are generally ±1 mm or less [9]. In these cases, the large curvature required of typical interventional flexible manipulators has the effect of amplifying the internal device friction which leads to greater hysteresis [4]. In addition, due to device mechanical constraints, these devices are unable to assume a small bending radii when articulating, further limiting their dexterity\(^1\). The limits in performance and dexterity of existing flexible catheter systems results in longer procedure times and may adversely affect patient outcomes.

A number of researchers have investigated alternative continuum design approaches, deviating from the tendon-actuated continuum thermoplastic designs found in the vast majority of commercially available flexible medical devices, such as catheters. In general, these approaches have sought to improve performance while maintaining the device's small size and ability to navigate complex paths. In [10-12] a novel concentric tube design is used to achieve a very small device cross-section, facilitating access to small anatomical features. In this case, while device compliance can be kept low, the fundamental trade-off between compliance (for safety) and performance still limits positioning accuracy. In [13-15], a highly articulated, redundant robot probe provides a high degree of maneuverability while maintaining the proximal shape of the probe and thus reducing the chance of injury to sensitive tissue. However, the design approaches in [13-15] employ relatively stiff and/or rigid-link construction, potentially compromising the inherent safety embodied by the compliant manipulator concept.

Recently, the use of feedback and associated sensing of flexible MIS robotic manipulators has been explored by a number of investigators to improve the performance of inherently safe flexible continuum manipulators. In [16] a closed-loop system was developed to control end-point position in both task space and joint space. Other examples include [17] where tracking of beating heart motion is explored, [10, 18] where concentric tube manipulators are controlled in position and end-point stiffness and [5, 13, 19-28] where various specialized control applications are investigated. Fundamentally, the inherent flexibility and internal friction of flexible medical continuum manipulators, such as cardiac intervention catheters, result in nonlinear hysteresis behavior that limits the closed-loop bandwidth. This, in turn, compromises the devices' ability to reject disturbances at the required time scales. In addition, the nonlinear, non-stationary motion characteristics of these compliant devices often result in limit cycling when used in closed-loop control, reducing the effectiveness of feedback approaches. This is particularly difficult to address for multi-degree-of-freedom manipulators where the hysteresis-induced nonlinear motion is complex and difficult to predict.

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\(^1\) A typical cardiac interventional flexible manipulator (i.e. catheter) ranges from 2 to 5 mm in diameter and is constructed of soft thermoplastics. Actuation is commonly achieved through a combination of control tendons, to affect catheter bending, and telescoping motion of successive catheter sections. Typical minimum bending radii range from 5 to 30 mm which may prohibit certain interventions in certain patients because the manipulator cannot access the necessary interventional targets.
2. An Interleaved Manipulation Approach

While the design and feedback approaches previously investigated have provided improvements in the performance of flexible continuum manipulators, none have achieved the performance levels typical of rigid-link designs while maintaining the compliant, atraumatic manipulator characteristics preferred for safety critical applications. The authors believe that the difficulty in achieving both inherent safety and performance is due to fundamental limitations that exist when working with flexible continuum manipulators.

To overcome these challenges we have proposed a new approach to continuum robotic manipulator design and actuation – where the safety advantages of flexible continuum manipulators are merged with the performance advantages of traditional rigid-link manipulators [29]. The approach advocates the combination of flexible, actively actuated continuum segments with small, rigid-link actuators. The small rigid-link joints are interleaved between successive continuum segments and provide a redundant motion capability. The authors refer to this approach as interleave continuum-rigid manipulation (see Figure 1). The active continuum segments provide large motion capability through, for example, a combination of tendon-driven articulation and telescoping motion. The compliant atraumatic construction of the continuum segments enhance safety while the small size of the rigid-link joints allows both the joint and limited stroke-actuator to be embedded inside the profile of the compliant segments. The limited stroke allows the rigid-link joints to assume a compact form, allowing for the use of a wide variety of micro-scale actuation concepts [30]. The repeatable, predictable motion of the small actuators allows for active correction of motion errors. The introduction of the small rigid-joints is central to the overall concept – in that they act as linearizing elements in a system whose overall behavior is highly nonlinear – thus allowing for effective use of feedback control to enhance performance.

![Figure 1: Conceptual overview of interleaved continuum-rigid manipulation](image-url)
The work presented here is focused on the discussion and evaluation of the interleaved approach introduced in [29]. Specifically, we present a guided discussion of the manipulator kinematics and control approach including an exploration of both the performance and dexterity improvements possible. In addition, the validation and evaluation of the interleaved approach has been augmented with additional experimental data and an improved multi-degree-of-freedom manipulator simulation. For clarity, some of the material presented in [29] is included here.

3. **KINEMATICS AND CONTROL APPROACH**

3.1 **Kinematics**

The overall manipulator kinematics description can be developed by considering the kinematics of the flexible segment and rigid-link joints separately. The flexible segment kinematics description is a function of the flexible segment actuation and design characteristics. For the purposes of this paper we will limit the discussion to flexible segment articulation only (via application of tension to control tendons) and exclude motions such as extension (from a proximal segment) and roll (relative to a proximal segment). Additionally, we assume that the flexible segment material behaves linearly in the range of strains to be considered. With these assumptions, we can adopt the kinematic description developed in [19, 31] where the flexible segment motions, or joint variables, are represented by the segment curvatures $\kappa_x$ and $\kappa_y$ representing the curvature in the x-z and y-z planes respectively, and the axial strain $\varepsilon_a$ (see Figure 2).

![Figure 2](image)

Assuming a consistent application of control tendon tension, these three joint variables are not independent. For the purposes of this discussion, we assume that the curvatures $\kappa_x$ and $\kappa_y$ are independently specified while the axial strain $\varepsilon_a$ is a dependent variable. This approach assumes that the articulation of the flexible segment results in constant curvature over the complete length of the segment. For this assumption to hold the effects of internal control tendon friction must be negligible as significant friction would cause the segment curvature to vary as a function of control tendon motion [4].
The kinematics of a single flexible segment can be represented using a homogeneous transformation $T_f$ whose elements are a function of the joint variables $(\kappa_x, \kappa_y, \epsilon_a)$ (see Figure 2(a))

$$T_f = \begin{bmatrix} R_f & \dot{P}_f \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (1)$$

The rotation matrix $R_f$ can be evaluated using the axis-angle representation [32] for a rotation $\alpha$ about a fixed axis $\hat{k}$.

$$R_f = \begin{bmatrix} k_x v_a + c_a & k_y v_a - k_z s_a & k_z v_a + k_y s_a \\ k_y v_a + k_z s_a & k_x v_a + c_a & k_z v_a - k_x s_a \\ k_z v_a - k_y s_a & k_y v_a - k_x s_a & k_x v_a + k_y c_a \end{bmatrix}. \quad (2)$$

where $c_\alpha = \cos \alpha$, $s_\alpha = \sin \alpha$, and $v_\alpha = 1 - \cos \alpha$.

The rotation magnitude $\alpha$, commonly referred to as the articulation angle, is given as

$$\alpha = \kappa L_f \quad (3)$$

where the length of the flexible segment $L_f$ and the total curvature $\kappa$ are given as

$$L_f = l_f(1 + \epsilon_a) \quad (4)$$

$$\kappa = \sqrt{\kappa_x^2 + \kappa_y^2}. \quad (5)$$

with $l_f$ representing the undeformed length of the flexible segment. The unit vector about which the rotation occurs is given as

$$\hat{k} = [k_x \ k_y \ k_z]^T = [-\sin \theta \ \cos \theta \ 0]^T \quad (6)$$

where roll angle $\theta$ is evaluated by

$$\theta = \tan^{-1}(\kappa_y / \kappa_x). \quad (7)$$

The position vector $\vec{P}_f$, describing the position of the end-point of the flexible segment, is given as

$$\vec{P}_f = \begin{bmatrix} x_f \\ y_f \\ z_f \end{bmatrix} = \frac{1}{\kappa} \begin{bmatrix} (1 - \cos \alpha) \cos \theta \\ (1 - \cos \alpha) \sin \theta \\ \sin \alpha \end{bmatrix}. \quad (8)$$

The rigid-link kinematics is a function of the specific joint mechanism design. For the purposes of this discussion, the rigid joint kinematics is represented by homogeneous transformation matrices $T_r$. The forward kinematics of the complete, interleaved continuum-rigid manipulator is assembled via the chain rule. When the flexible and rigid-link degrees-of-freedom are successively alternated, the complete manipulator forward kinematics for a manipulator with $n$ degrees of freedom is given as

$$T = (T_r)_1 (T_f)_1 \ ... \ (T_r)_n (T_f)_n. \quad (9)$$

In this case, the rigid-joint is assumed to be proximal to the corresponding flexible segment to leverage the larger workspace of the successive flexible segment. In addition to the forward kinematics, the
control approach, discussed in Section 3.2, is based on the instantaneous kinematics of the manipulator, which requires the Jacobian relating the flexible segment and rigid-link joint velocities to task space velocities. In this case, it proves convenient to form the Jacobian numerically using the forward kinematics discussed previously, where the elements of $J$ are the partial derivatives of task motions with respect to joint motions. The task-space Jacobian is represented by $J$ and is partitioned between flexible segment and rigid-link motions

$$J = [J_f | J_r].$$  \hspace{1cm} (10)

### 3.2 Control Approach

One of the central challenges of the interleaved approach is formulating an effective control strategy. There have been many formal methods developed for multi-input-multi-output control system design including $\mu$-synthesis, $H_{\infty}$, and, more recently, design approaches developed for dual-input-single-output system, such as the PQ approach [33]. In this application the rigid-link and flexible segment are not completely redundant actuators. While the manipulability of these two actuators must overlap, we expect that the rigid-link will generally be of greater precision, have less actuation range, and possibly be faster than the flexible segment actuator. These actuator differences, in addition to the nonlinear properties of the catheter, suggest a parallel control structure which explicitly partitions the task error signal, $\Delta x$, into high and low frequency signals. In the context of the overall control structure proposed (see Figure 3), the flexible segment and rigid-link task-space controllers ($D_f(s)$ and $D_r(s)$, respectively) perform this partitioning function as well as help shape the actuator closed-loop dynamics. Additionally, a task-space loop compensation block, $D_l(s)$ is included to compensate the additional dynamics that result from the parallel path summation of the rigid and flexible segment control signals. The high/low frequency partitioning is motivated by the desire to limit the motion of the limited-stroke rigid-link joints while correcting for motion errors that result from the slower-responding flexible segments.

![Figure 3: Overview of a candidate interleaved continuum rigid manipulator control structure.](image)

The flexible segment control includes a feed-forward inverse kinematics block which converts the desired task space configuration to flexible segment joint commands (i.e. segment curvatures). For the two degree-of-freedom simulation discussed in Section 4.20 as well as the one degree of freedom experimental testbed discussed in Section 4.1, the inverse kinematics pertaining to the coupled motion of the flexible sections (exclusive of rigid-link joint motion) are obtained using a multivariable Newton's method. The Jacobian $J_r$ relating flexible segment joint velocities to the task-space velocities is used in the iterative solver. As shown in Figure 3, the task space control signal is transformed to joint space...
motion commands via the flexible segment and rigid-link joint Jacobians, $J_f$ and $J_r$, under the assumption that the task space error is small.

It should also be noted that both the flexible segment Jacobian $J_f$ and rigid-link joint Jacobian $J_r$ are functions of the manipulator's configuration. As a result, knowledge of the configuration is required—either through estimation or direct measurement. While the rigid-link joint positions will likely closely track the desired rigid-link motions, the flexible segments are expected to have significant error and thus direct measurement of their motion is required to properly form the Jacobian for both the flexible segments and rigid-link joints.

While the specific structure of the compensation blocks ($D_f(s)$, $D_r(s)$, and $D_l(s)$ in Figure 3) can vary depending on the specific dynamics of the system under consideration and the desired performance goals, there are general considerations for both the flexible segment and rigid-link control that have bearing on the compensator design. In general, robotic catheter systems regulate control tendon motion with a high-gain position controller that acts on the control tendon actuator positions. This is done to improve disturbance rejection by increasing the static stiffness at the control tendon output and to improve stability margins. In addition, the low torque density of electromagnetic actuators (used almost exclusively in the type of cardiac interventional catheters under consideration here) generally requires the use of a gear reducer. The resulting increase in reflected inertia and friction amplification makes tension control difficult to implement in a robust manner. As such, it is assumed that the local joint controllers (i.e. tendon position and rigid-link joint position) are designed to have significantly faster closed-loop dynamics than those of the overall closed-loop interleaved manipulator. When considering the design of the compensation we can assume that the control inputs to the interleaved manipulator are given in terms of joint displacements ($q_f$ and $q_r$).

To compensate for steady-state flexible segment motion errors, integral control (and variants thereof such as lag compensation) has been successfully applied in the reduction of catheter kinematic errors [16]. In the context of the interleaved control structure proposed here, the flexible segment compensation block, $D_f(s)$, can assume a similar integral-like control structure (i.e. $K_i/s$). As described in [33], the ratio of joint control compensators ($D_{pr}(s) = D_f(s)/D_r(s)$) can be used to examine the frequency partitioning characteristics of the chosen compensator design. Assuming that the magnitude of $D_{pr}(s)$ decreases with increasing frequency, the crossover frequency, $\omega_{cr}$ of $D_{pr}(s)$ is the point where the magnitude of $D_f(s)$ and $D_r(s)$ are equal and thus the frequency at which the low and high frequency partitioning of the control input occurs. In addition, as described in [33], the phase of $D_{pr}(s)$ at the crossover is a representation of the constructive interference between the flexible segment and rigid-link joint control action. In this paper, where $D_f(s)$ was given an integral-like control structure, a suitable choice for the rigid-link joint controller could be unity gain (i.e. $D_r(s) = 1$).

As described earlier, the purpose of the overall loop compensation, $D_l(s)$, is to help shape the open-loop system frequency response such that the closed-loop stability margins and performance are satisfactory. In the simplified case where $D_f(s)$ is a simple integral controller and $D_r(s)$ is a unity gain, and when we assume that the rigid-link and flexible segment system plant transfer functions have constant gain and no

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2 The use of feedback in robotic catheter systems is motivated primarily by the desire to correct for device kinematic errors and to a lesser degree for the rejection of unmodeled disturbances. In closed-loop flexible robotic catheter applications, the modification of the system poles is not typically a design objective and, in general, is difficult to achieve due to the compliant drive train associated with control tendon actuation. While the direct manipulation of the system poles through state-feedback is theoretically possible, the ability to reduce steady-state errors is limited by the presence of the flexible body modes. In this case, the use of integral control is commonly employed to eliminate steady state errors while having no or limited effect of the system’s flexible mode roots.
phase distortion at low frequencies (i.e. \( G_f(s) = G_r(s) = 1 \)), a reasonable choice for \( D_l(s) \) could be a simple integral compensator – where the gain is adjusted to obtain a stable parallel system, presumably with a closed-loop bandwidth which is greater than the crossover frequency of \( D_{fr}(s) \) (i.e. the partitioning frequency). The control partitioning can be seen by examining the frequency response of the closed-loop system of the simplified case described above. As shown in Figure 4(a), the control signals associated with the flexible segment and rigid-link response are partitioned at the crossover frequency, \( \omega_{f/r} \), the point at which their respective magnitudes are equal. In addition, the high-frequency content from the rigid-link response results in a combined system closed-loop bandwidth, \( \omega_{CL} \) that is well above the crossover frequency, \( \omega_{f/r} \).

![Figure 4: Closed-loop frequency response of a one degree-of-freedom simplified interleaved system. (a) magnitude and phase as a function of frequency – where the contribution of the flexible segment actuation, rigid-link actuation, and the combination of the two are shown; (b) frequency response – shown as real and imaginary part of response.](image)

4. Evaluation

The performance of the interleaved continuum rigid manipulator approach described above was evaluated experimentally, using a one degree of freedom validation testbed, and through simulation, using a two degree of freedom planar manipulator simulation. The results of this evaluation are presented in the following sections.

4.1 Experimental Performance Validation

4.1.1 Experimental Testbed Overview

A one degree-of-freedom testbed was used to investigate the performance characteristics of the interleaved continuum-rigid manipulation approach. An overview of the testbed is shown in Figure 5. The testbed manipulator consists of a single articulating (i.e. bending) flexible-segment and a proximal rigid-link revolute joint. The flexible-segment consists of a 6.4 mm diameter urethane body which is articulated using a pair of opposing control tendons (Spectra fiber, 0.23 mm diameter) anchored at its distal end. The control tendons are actuated by a pair of DC brush gear motors (Maxon Motor, GmbH), the positions of which are controlled with a high-bandwidth (~35 Hz closed-loop bandwidth) position controller. Actuation of the tendons causes the flexible segment to articulate within a vertical plane through the kinematics discussed in Section 3.10. The rigid-link joint motion provides rotation about a pivot axis located at the base of the flexible segment which is perpendicular to the flexible segment actuation plane. The flexible segment control tendons intersect the rotation axis of the rigid-link to
eliminate coupling between the flexible and rigid-link motions. The rigid-link joint rotation is accomplished through a slider-crank mechanism actuated by a voice-coil motor (BEI Kimco Magnetics) with approximately 6 mm of travel – which results in approximately ±16 degrees of manipulator articulation. Catheter tip motion is acquired with an Ascension trakStar 3D magnetic position sensor, operating at approximately 200 Hz, providing a globally-referenced measurement of the catheter’s tip position. The controller is implemented using Matlab xPC 2009a (Mathworks, Natick, Massachusetts, U.S.A). As mentioned above, our present focus is on investigating the advantages of interleaved actuation. The testbed is not intended as a design prototype and, as such, we have made no attempt at rigid-link miniaturization.

The interleaved control structure implemented on the experimental prototype is identical to the one described in Section 3.2 (see Figure 3). In this case, the frequency partitioning between the flexible and rigid-link joints was adjusted via the flexible segment and rigid-link compensation blocks, $D_f(s)$ and $D_r(s)$, respectively. As described in Section 3.2, $D_r(s)$ was set equal to 1.0 while $D_f(s)$ was assigned a simple integral control structure (i.e. $D_f(s) = \frac{K}{s}$). The integral gain of $D_f(s)$ was adjusted to have a crossover frequency of 0.05 Hz, below which the flexible segment primarily acts on the task error and above which the rigid-link primarily acts on the task error. The choice of this partitioning frequency is essentially limited by the stability of the flexible segment, which, in addition to the catheter and actuator characteristics, is a function of tendon compliance and friction developed throughout the catheter body. The partitioning frequency must therefore be conservative to allow for variable friction and compliance.
during actuation; the experimental partitioning frequency of 0.05 Hz is the result of this balance and is consistent with the authors’ experience. The task-space loop compensation block, \( D_I(s) \), is given as an integral controller to eliminate steady-state errors due to internal device friction and kinematic modeling errors. The integral gain of the task-space compensation block was adjusted upward until signs of instability were observed, resulting in an overall system open-loop compensated crossover frequency of approximately 0.6 Hz. This crossover frequency is chosen to be below the catheter’s first mode of ~1.8 Hz and maintain sufficient stability margins.

### 4.1.2 Performance Evaluation

To evaluate performance, the responses of the interleaved system and a system consisting of only a flexible-segment were compared. The flexible segment-only system was formed by preventing rigid-link joint motion while using the same flexible segment as the interleaved system. To provide a clear comparison of the control behavior of each system, the flexible segment feed-forward term, shown in Figure 3, was set equal to zero in both the interleaved and flexible-segment only implementations.

The performance of the two systems was evaluated with a simple step response with the task defined by the catheter tip articulation (see Figure 5(b)). In the first experiment, the manipulator was positioned approximately in the center of its workspace (vertical) and biased slightly positively (~11°) to eliminate any effects of control tendon slack. A small effective articulation motion step input command (~17°) was applied and the position control performance was measured. Figure 6 shows the results of this first, low articulation experiment. As seen in Figure 6(a), the response of the interleaved system is approximately three times faster than the flexible-segment only closed loop system – due primarily to the ability of the rigid-link joint to effect changes in articulation faster than the more compliant flexible-segment control tendons allow.

![Figure 6: Experimental response of the interleaved experimental testbed to a commanded step end-point articulation of 17 degrees from an initial articulation of 11.5 degrees. (a) Response of interleaved system (red) and response of flexible-segment only manipulator (blue); (b) total response of the interleaved system (red, same as in (a)), estimated flexible segment contribution (blue), and estimated rigid-link contribution (green).](image)

To gain a better understanding of the interleaved response, Figure 6(b) shows a projection of actuator encoder data into the task space to estimate the contribution of each actuator to the total tip articulation. Note that this projection does not include catheter dynamics and therefore has some inherent error. Nevertheless, Figure 6(b) clearly shows the effect of the frequency partitioning between the slower flexible segment and faster rigid-link joint controllers. The response of the rigid-link joint actuator is
almost immediate, reacting to the high-frequency content contained in the step input command. As the slower flexible segment actuator motion increases, the rigid-link joint actuator motion decreases in magnitude – returning to the center of its actuation range. The summation of the two results in a faster response as compared to the flexible-segment alone.

Figure 7: Experimental response of the interleaved experimental testbed starting from a large initial end-point articulation. Response of the system to a commanded step end-point articulation of 17 degrees from an initial end-point articulation of 218 degrees. (a) Response of interleaved system (red) and response of flexible-segment only manipulator (blue), (b) response of interleaved system (red), estimated flexible segment contribution (blue), and estimated rigid-link contribution (green).

In a second experiment, the same step command was applied (~17°) with the manipulator initially positioned with an equivalent articulation of 218° (i.e. in a U-shaped initial configuration). As seen in Figure 7(a), the response of the interleaved system suffers from substantial overshoot – due primarily to the saturation of the rigid-link joint. To gain a better understanding of the interleaved response, Figure 7(b) shows a projection of actuator encoder data into the task space to estimate the contribution of each actuator to the total tip articulation. Figure 7(b) clearly shows the saturation of the rigid-link joint motion. Note that the saturation of the rigid-link joint results in the overshoot of the flexible segment motion command and the corresponding effect on the overall system response.

To understand the source of the response overshoot and explore the effects of rigid-link joint saturation on overall system stability, a series of similar step response experiments were carried out – where limitations on the rigid-link joint range of motion were emulated by restricting commands to the rigid-link position controller. The experimental step response for various levels of emulated rigid-link joint range of motion is shown in Figure 8. As seen in Figure 8, as the rigid-link joint range of motion is reduced, the response becomes more oscillatory. In the extreme case, where the rigid-link joint motion is set equal to zero, the system is only marginally stable.

The reduction in system stability (due to rigid-link joint saturation) can be understood by examining the system’s stability margins as a function of joint saturation levels. To do this, we construct a model of the compensated open-loop system which is consistent with experimental frequency response measurements and examine the effects of rigid-link joint saturation by varying the joint gain (as an approximation to saturation \([34]\)) (see Figure 9). The overall system loop stability as a function of rigid-link joint saturation can be determined by evaluating the phase and gain margin of the compensated open-loop system (Figure 9) as the saturation gain, \(K_s\), is varied from 1.0 (no saturation) to 0.0 (total saturation).
Figure 8: Effect of limited rigid-link joint range of motion (position saturation) on controller performance. Shown above are the step responses of the interleaved testbed for various levels of rigid-link joint saturation. Joint saturation was emulated by limiting the commanded position input magnitude in software.

Figure 9: Model of the compensated open-loop interleaved system. Rigid-link saturation is approximated with a variable gain, $K_s$ [34].

As shown in Figure 10, the nominal compensated open-loop interleaved system (with no saturation) has closed-loop phase and gain margins of approximately 80 degrees and 0.6 dB, respectively. The nominal system stability margins reflect the design objective where by the highest possible closed-loop bandwidth was sought. Here the closed loop bandwidth is restricted by the first flexible mode of the flexible segment. The lightly damped mode is responsible for the low gain margin while the integral control action results in a robust phase margin in excess of 80 degrees. As the rigid-link joint saturation increases (approximated by a reduction in $K_s$), the resulting system phase margin decreases – resulting in a closed-loop response which is lightly damped. At the extreme, where the saturation is total (i.e. no rigid-link joint motion), the system’s phase margin approaches zero – resulting in a highly oscillatory response. In practice, unmodeled system dynamics would lead to an unstable system. As such, the effects of saturation should be carefully considered in the design of any interleaved system control implementation.
4.2 Simulation Performance Validation

To supplement the single degree-of-freedom experimental validation, a two degree-of-freedom planar manipulator simulation was developed to investigate the interleaved approach in the context of a multi-degree-of-freedom system. In the simulation, the flexible segments are modeled by a serial chain of links constrained by revolute joints (see Figure 11). Flexible segment bending compliance and internal damping were modeled with parallel linear torsional springs and dampers which act across the revolute joint. Flexible segment control inputs, applied via prescribed proximal control tendon motions, acting across the tendon stiffness, are applied as torques at the revolute joints where the tension magnitude and local curvature determine the magnitude of the applied torques. To model the effects of internal control tendon friction, which can have a significant effect on flexible segment motion, a modified Dahl friction model was used – whereby the steady-state Dahl friction torque is related to control tendon tension as well as local flexible segment curvature [4]. The Dahl friction forces are applied as forces at the tendon-segment sliding interface.

The rigid-link joints are modeled as revolute joints, the input of which imposes a displacement between successive flexible segments. The implicit assumption being that the rigid-link joints have output impedance that is sufficiently high such that the dynamics of the flexible segments have negligible effect on the relative position of the rigid-link joints. In addition, the simulation assumes that the rigid-links are designed so that the flexible segment control tendon tension and rigid-link joint motion are uncoupled. This uncoupling can be achieved by routing the control tendons across the rigid-link joint rotation axes such that joint motion does not result in a control tendon length change - resulting in no work being done to the system.

The control structure implemented in the simulation is identical to the approach described in Section 3.2 (see Figure 3). In this case, the rigid-link compensation, $D_r(s)$, was set to unity gain while the flexible-segment compensation block, $D_f(s)$, were set to a simple integral filter ($D_f(s) = \frac{K_f}{s}$). The gain, $K_f$, was adjusted to obtain a partitioning frequency (between the flexible and rigid actuation) of approximately 0.1 Hz. The loop compensation, $D_l(s)$, was set to a simple integral filter ($D_l(s) = \frac{K_l}{s}$) where $K_l$ was adjusted to obtain the highest possible closed-loop bandwidth – approximately equal to 1.0 Hz within the

Figure 10: Interleaved system stability margins as a function of joint saturation levels. The degradation of the overall system stability (as evidenced by the reduction in phase margin) is shown in the bode plot (left) and the supporting phase-gain plot (right).
workspace of interest. The closed-loop bandwidth is primarily limited by the 1st flexible mode of the two-segment manipulator.

Figure 11: Overview of two degree-of-freedom planar simulation model.

The planar simulation described above was used to evaluate the performance of the interleaved continuum-rigid manipulation approach in the context of a multi-degree-of-freedom system. In the simulation experiment, the manipulator end-point was commanded along a square trajectory of width 10 cm (see Figure 12). The end-point velocity profile followed a haversine function with a wavelength equal to the width of the square motion profile. The use of a haversine function guaranteed that the linear end-point acceleration was a continuous function over the complete trajectory.

Figure 12: Commanded end-point square trajectory and resulting two-segment manipulator configuration. (a) two segment manipulator model in initial configuration, (b) end-point trajectory and associated manipulator configurations.
Figure 13: Simulated tracking performance of a two-degree-of-freedom interleaved manipulator over a range of tracking speeds. The magnitude of the end-point velocity (haversine) profile was varied from 5 cm/s (in (a)) to 40 cm/s (in (d)).

The simulated tracking results are shown in Figure 13 for various tracking speeds. As seen in Figure 13(a), the closed loop two-segment interleaved system tracks the square trajectory with error of less than 0.25 cm while the uncompensated flexible-segment manipulator error exceeds 2 cm. In this case, the open-loop errors are due to a combination of internal control wire friction and uncertainty in the open-loop flexible segment kinematics. As the tracking speed is increased, the frequency content in the commanded trajectory increases. When the frequency content exceeds the closed-loop bandwidth of the position controller, the tracking performance degrades – as is seen in Figure 13(b) and (c). As such, the mechanical design of an interleaved manipulator should consider strategies which allow for a higher closed-loop bandwidth – including designs which increase the flexible mode frequencies of the uncontrolled device.

4.3 Kinematic Design Considerations

The kinematic design and specific device mechanical design details will have a significant effect on the performance of the manipulator. It is useful to explore both in the context of the interleaved approach.
As described earlier, one of the functions of the limited-stroke rigid-link joints is to compensate for flexible segments motion errors. As such, the task space motion bounds of the rigid-link joints should envelope the task-space error bounds of the flexible segments.

The task space error can be evaluated as:

$$\Delta x_f \cong J_f \Delta q_f$$  \hspace{1cm} (11)

where $J_f$ is the flexible segment Jacobian and $\Delta q_f$ is the flexible segment joint space motion errors. The task space error, $\Delta x_f$, at a given configuration is evaluated by mapping the joint space error bounds to task space using equation (11). Similarly, the task space motion due to the motion of the rigid-link joints can be evaluated as:

$$\Delta x_r \cong J_r \Delta q_r$$  \hspace{1cm} (12)

where $J_r$ is the rigid-link joint Jacobian and $\Delta q_r$ is the rigid-link task space motion. Using (11) and (12), the flexible segment task-space error bounds can be evaluated from the joint space error bounds and the rigid-link task space motion bounds can be evaluated from the rigid-link joint limits (see Figure 14). By comparing the error and motion bounds, we can evaluate the regions where the motion error due to the flexible segments can and cannot be corrected by the rigid-link joint motion (see Figure 14).

Figure 14: Flexible segment and rigid-link task-space error and motion bounds (for a two degree-of-freedom manipulator).

An example two degree of freedom manipulator, overlaid with the error and motion bounds of the flexible and rigid-link joints respectively, is shown in Figure 15. The regions of uncorrectable error are a function of the rigid-link joints' range of motion, the error ranges of the flexible segments, and the configuration of the manipulator.

Alternatively, we can evaluate the rigid-link joint motions required to fully correct for the flexible segment motion errors. In this case, the required rigid-link joint motions are evaluated by equating equations (12) and (13) and solving for $\Delta q_r$.

$$\Delta q_r \cong J_r^{-1} J_f \Delta q_f$$  \hspace{1cm} (14)
To evaluate the required rigid-link joint motion to compensate for all possible flexible segment errors at a given configuration, equation (14) can be used to map the set of flexible segment joint errors limits to the set of corresponding required rigid-link joint motions – the maximum of which corresponds to the required rigid-link joint range of motion (ROM) to compensate for all possible errors at the specified configuration (see Figure 16).

Figure 15: Task-space error and motion bounds of the flexible segment and rigid-link joints. The proximal and distal flexible segment articulation errors depicted are ±0.15 radians. The rigid-link joint range of motion depicted are ±0.10 radians.

Figure 16: Evaluation of required rigid-link joint motion to correct for flexible segment motion errors. Using the two degree-of-freedom example introduced previously, we can see how the required rigid-link joint ROM varies as a function of manipulator configuration. In this case, we make the assumption that the flexible segment joint motion errors (defined as deviations in the flexible segment curvature) are
proportional to the magnitude of the flexible segment curvature. This assumption is based on prior catheter modeling and experimental data given in [4] and is due to the increased tendon friction forces (and resulting error in curvature) that occur with high segment curvature. As shown in Figure 17, the required rigid-link ROM (to fully compensate for the flexible segment errors) is a strong function of segment curvature and thus varies considerably over the workspace of the device. The required ROM is largest in configurations where both segments have high curvature – resulting in large flexible segment errors.

In Figure 17, the required ROM for the rigid-link joint #2 is generally larger than the ROM for joint #1. However, as the manipulator’s nominal configuration is varied the rigid-link’s required ROM varies as well. For instance, if the nominal position of the rigid-link joint #2 is set to 90 degrees, then the required rigid-link joint ROM (to compensate for flexible segment errors) changes significantly (see Figure 18). In this case, the required rigid-joint ROM is generally less than that shown in the prior example, where the nominal rigid-link joint positions were set to 0 degrees (Figure 17). However, with this arrangement, the defined workspace contains a singularity in the rigid-link joint motion Jacobian, \( J_r \). As seen in Figure 18(b) and (c), the required rigid-link joint ROM increases significantly in the vicinity of the singularity (lower left of the configuration space). As seen in Figure 19, the rigid-link joint motion bounds are reduced to a single dimension along the curve of rigid-link joint singular positions and thus, make compensation of the flexible segment motion errors (via rigid-link joint motion) impossible. From these two examples it is clear that the kinematic arrangement chosen will have a significant impact on the achievable performance improvements (in regards to error correction via rigid-link joint motions).

Figure 17: Required minimum rigid-link joints range of motion (ROM) to fully compensate for flexible segment motion errors as a function of manipulator configuration. The nominal angle of the rigid-link joints is set equal to zero and the flexible segment joint limits, given in segment curvature, are [0 to 0.5] and [0 to 0.6] cm for segments #1 and #2, respectively ([0° to 90°] and [0° to 170°] of articulation. The flexible segment joint motion errors are assumed to be proportional to the magnitude of the flexible segment joint motions (in this case we assume a ±20% variation of flexible segment curvature). (a) Two degree-of-freedom interleaved manipulator (flexible segment) workspace. (b) Contour plot of rigid-link joints #1 required ROM over complete workspace. (c) Contour plot of rigid-link joints #2 required ROM over complete workspace.
Figure 18: Required minimum rigid-link joints range of motion (ROM) to fully compensate for flexible segment motion errors. The nominal angle of the rigid-link joint #1 and #2 are set equal to 0.0 and 90°, respectively. The flexible segment joint motion errors are assumed to be proportional to the magnitude of the flexible segment joint motions (in this case we assume a ±20% variation of flexible segment curvature). (a) Two degree-of-freedom interleaved manipulator (flexible segment) workspace. (b) Contour plot of rigid-link joints #1 required ROM over complete workspace. (c) Contour plot of rigid-link joints #2 required ROM over complete workspace.

Figure 19: Flexible segment error bounds and rigid-link joint motion bounds drawn along the curve of rigid-link joint singular configurations.

In addition to performance improvements, the redundant rigid-link joint motion has the potential to increase the overall manipulator dexterity. The primary limitation on the dexterity of flexible manipulators such as robotic catheters is the limited curvature that the structure can assume – above which the flexible segment can experience mechanical failure. With the introduction of interleaved rigid-link joints, this limitation can be overcome. While the design space is complex and direct comparison of a flexible-only manipulator to an interleaved design is difficult, it is still instructive to examine the dexterous workspace of a simple planar manipulator. In this example, the flexible segment-only manipulator consists of three equal length serial flexible segments. The total length of the manipulator is 150 mm and the minimum possible flexible segment radius is limited to 45 mm (equivalent to a maximum curvature of 0.022 1/mm). The interleaved manipulator consists of two equal length flexible segments with a single rigid-link joint (with 90° nominal orientation) interleaved between the proximal and distal flexible segments. The total length of the manipulator and the minimum flexible segment radius are the same as for the flexible-segment only manipulator. In this case, the task is defined by the position of the manipulator end point and orientation of the distal end of the most distal segment. To
limit the scope of the analysis, the task is constrained to maintain a horizontal tip orientation. Given the task constraint on orientation, the dexterous workspace of the flexible segment-only manipulator is shown in Figure 20(a).

![Figure 20: Comparison of dexterous workspace for a simple example three-degree-of-freedom manipulator. (a) Dexterous workspace for example 3-segment flexible manipulator. (b) Dexterous workspace for an example interleaved manipulator (2 flexible segments and one interleaved rigid-link joint) as a function of rigid-link joint range of motion (ROM).](image)

In comparison, the dexterous workspace of the example interleaved manipulator is shown in Figure 20(b). As seen in Figure 20(b), the workspace is a function of the joint range of the rigid-link joint. For modest rigid-link joint motions (<45°), the dexterous workspace area is comparable to that of the flexible segment-only manipulator. However, as the rigid-link joint range of motion is increased, the dexterous workspace increases significantly. When the rigid-link joint range of motion is equal to ±90°, the area of the interleaved manipulator’s workspace is almost three times larger than the workspace of the flexible segment only. If the rigid-link joint range of motion is increased to ±180°, the ratio of interleaved to flexible segment only workspace area increases to more than 5.

While this is just a representative example (with no consideration for design optimization in regards to relative segment lengths) it does serve to illustrate the potential of the interleaved approach in regards to end-effector dexterity. From the two examples described, it is clear that the effectiveness of the approach is dependent on the kinematic arrangement selected while the requirements on rigid-link joint range of motion are directly linked to the kinematic design choices made.

5. Summary

To overcome the performance challenges of using flexible continuum manipulators in minimally-invasive surgical procedures we have described a new approach to continuum robotic manipulator design and actuation that combines flexible, actively actuated continuum segments with small, rigid-link joints – retaining the safety advantages of flexible continuum manipulators while achieving the performance advantages of traditional rigid manipulators. The performance improvements of the proposed approach have been demonstrated both experimentally with a one degree-of-freedom testbed and in simulation with a two degree-of-freedom planar model. In regards to improvements in response speed – due to an increase in the manipulator’s closed-loop position bandwidth – the benefits of the approach are most evident in the case where there is an extended proximal flexible section (as would be present in a device intended for vascular access). In this case, distally-located rigid-link joints can bypass the compliance and friction of the proximal flexible section, resulting in faster closed-loop response. Dexterity
improvements, such as an increase in the dexterous workspace, are dependent on both the kinematic arrangement of the manipulator and the range of motion of both the flexible and rigid-link joints. Future work will focus on the development of clinical-ready device prototypes suitable for animal model evaluation, including the miniaturization of the rigid-link joint and actuation design.

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